MATHCOUNTS®

2021 PRACTICE COMPETITION 1

▽ Sprint Round 1–15

▽ Target Round 1–4

▽ Team Round 1–5

▽ Answer Key

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PLEASE NOTE:

For this practice competition, students were given the same amount of time as they will have on the official Chapter Competition, but it included only half the number of Sprint, Target and Team Round problems found on an official competition.

The Individual Score is comprised of a student's Sprint and Target scores. With fewer problems, the maximum Individual Score for this practice competition is 15 + 2 × 4 = 23 points. The maximum Individual Score on the official Chapter Competition will be 30 + 2 × 8 = 46 points.

Raytheon Technologies

2021 MATHCOUNTS
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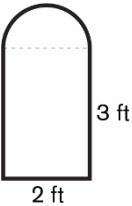
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Sprint 1 The following equation was written on the chalkboard: $\Delta + \Delta + \Diamond + \Theta - \Delta = \Omega$. If $\Delta = 4$, $\Theta = 3$ and $\Omega = 12$, what is the value of $\Diamond$?

Sprint 2 Two integers have a difference of 18 and a sum of 2. What is the product of the two integers?

Sprint 3 A map of Wyoming is drawn with a scale of 1/4 inch = 1 mile. On that map, how many inches long would the drawing be of a road that is 10 miles long? Express your answer as a decimal to the nearest tenth.

Sprint 4 The median of a set of consecutive odd integers is 138. If the greatest integer in the set is 145, what is the least integer in the set?

Sprint 5 A jar contains only nickels, dimes and quarters. There is at least one of each type of coin in the jar. If the total value of the coins in the jar equals 60 cents, how many quarters are in the jar?

Sprint 6 After all markdowns and discounts, Charlene's prom dress cost her $22 before tax. The dress was on a rack labeled "50% off lowest marked price," and the lowest marked price was already a 75% reduction from the original price. What was the original price of the dress?

Sprint 7 A Norman window consists of a rectangular region topped by a semi-circular region. How many square feet are in the area of the glass needed to fill the two regions of a Norman window whose rectangular region measures 2 feet by 3 feet, as shown? Express your answer to the nearest whole number.

Sprint 8 Two rectangular boxes have the same volume. One box is a cube, and the other box has dimensions 8 ft by 4 ft by 16 ft. How many feet long is an edge of the cube?

Sprint 9 If a standard six-sided die is rolled twice, what is the probability that the second number rolled is not less than the first number rolled? Express your answer as a common fraction.

Sprint 10 Eight toy camels and three toy pigs cost Gary $85. Twelve toy camels cost Sherry $96. Assuming everyone bought their toys at the same store and there were no discounts, what is the cost of two toy pigs?

Sprint 11 How many 4-letter "words" can be made using all the letters in BEEP, if the two Es are indistinguishable? Two such "words" to include are BEEP and EBEP.

Sprint 12 In an effort to reduce accidents, the highway authority decided to reduce the speed limit on a road from 65 mi/h to 50 mi/h. To travel 650 miles, how many more hours will it take a car averaging 50 mi/h than a car averaging 65 mi/h?

Sprint 13 A bag contains 8 white marbles, 6 red marbles and 4 blue marbles. How many blue marbles must be added to the bag so that the probability of choosing a blue marble is 1/2?

Sprint 14 The ratio of 6th graders to 7th graders in a class of 35 students is 3:4. How many more 7th graders than 6th graders are in the class?

Sprint 15 The heights of six students Aaron, Betty, Carrie, Dan, Evelyn and Frank are 58 inches, 60 inches, 63 inches, 64 inches, 68 inches and 69 inches. Carrie is 4 inches shorter than Aaron. The girls (Betty, Carrie and Evelyn) are the three shortest students. Dan is one inch shorter than Frank. Betty is the shortest student. What is the sum of Frank's height and Evelyn's height, in inches?

Target 1   Tia decided to use her average cost for utilities last year to project her expenses for the future. Last year, she spent an average of $216 per month on utilities, but she anticipates a 5% increase in the annual cost of utilities. Based on this information, how much should she expect to pay for utilities each month this year?

Target 2   John takes one step every second, and each step is 33 inches long. How many miles per hour is John walking, given that 5280 feet = 1 mile? Express your answer as a decimal to the nearest thousandth.

Target 3   Captain Hook, Captain Sparrow and Long John Silver split a treasure in the ratio 2:5:7. Long John Silver received the greatest portion. Captain Hook received the smallest portion, which was 1000 pounds. How many pounds was the total weight of Long John Silver's portion of the treasure?

Target 4   A block of cheese measuring 5 inches by 2 inches by 3 inches is coated in a very thin layer of wax. The cheese is sliced into 240 cubes with half-inch edges. How many cubes will not have any wax on them?

Team 1   Mr. Smith has a monthly income of $4875. He budgets $1625 a month for rent, $975 for child care, $975 for savings, $975 for food and utilities, and the rest for entertainment. Mr. Smith constructs a circle graph, like the one shown here, to represent his monthly budget. How many degrees are in the central angle of the sector representing entertainment?
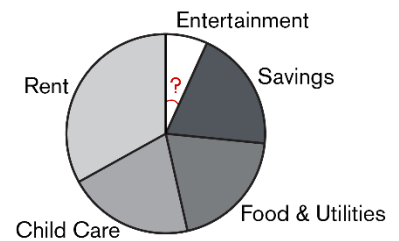
Team 2   In the 2018-2019 season, the weekly salaries of musicians in major orchestras ranged from $734 to $1925. Orchestras have different season lengths ranging from 24 weeks to 52 weeks. In one orchestra season, what is the positive difference between the most that could be earned and the least that could be earned in a year?

Team 3   Alicia recorded the statistics shown at last night's basketball game. The percents were rounded to the nearest whole number, and only whole numbers of shots could be successful. What percent of the 45 attempted shots were successful?

	2-Point Shots		3-Point Shots	
Player	**Attempted**	**% Successful**	**Attempted**	**% Successful**
Lisette	15	67%	5	40%
Sara	10	80%	2	50%
Jen	4	75%	1	0%
Tai	6	33%	2	50%

Team 4   A standard deck of 52 playing cards consists of four different suits and 13 cards of different ranks in each suit. Four cards will be drawn at random without replacement from this deck. The probability that the four cards will be all the same suit is k times the probability that they will be all the same rank. What is the value of k?

Team 5   A cylindrical can contains three tennis balls. The diameter of each tennis ball is 8 cm. If the tennis balls fit snugly against the interior of the can and against the top and bottom of the can, as shown, how many cubic centimeters of empty space are in the can? Express your answer in terms of π. (Hint: The formula for the volume of a sphere is $V = (4/3)\pi r^3$, and the formula for the volume of a cylinder is $V = \pi r^2 h$.)

2021 PRACTICE COMPETITION 1 — Answers

Sprint 1	5	Target 1	226.80	Team 1	24
Sprint 2	−80	Target 2	1.875	Team 2	82,484 or 82,484.00
Sprint 3	2.5	Target 3	3500	Team 3	60
Sprint 4	131	Target 4	64	Team 4	220
Sprint 5	1			Team 5	128π
Sprint 6	176 or 176.00				
Sprint 7	8				
Sprint 8	8				
Sprint 9	7/12				
Sprint 10	14 or 14.00				
Sprint 11	12				
Sprint 12	3				
Sprint 13	10				
Sprint 14	5				
Sprint 15	132				

Sprint 1

When the values are substituted into the equation, we have 4 + 4 + ◊ + 3 − 4 = 12. This simplifies to 7 + ◊ = 12, so ◊ = **5**.

Sprint 2

Let x and y represent the two integers. We have $x + y = 18$ and $x − y = 2$. Adding these equations, we get $2x = 20$, so $x = 20/2 = 10$. Substituting this value into the equation $x + y = 18$ gives us $10 + y = 2$, so $y = 2 − 10 = −8$. The product of the two integers is $xy = 10 × (−8) =$ **−80**.

Sprint 3

Since each mile is 1/4 inch, 10 miles would be 10 × 1/4 = 10/4 inches or **2.5** inches.

Sprint 4

This is a set of consecutive odd integers that has a median of 138, which is not odd. The median, 138, must be the mean of the two middle integers of the set, and the two middle numbers must be 137 and 139. Since 145 is the greatest integer, the set contains four integers greater than the median, namely 139, 141, 143 and 145. There must also be four integers in the set that are less than the median, namely 137, 135, 133 and 131. The least integer in the set is **131**.

Sprint 5

The jar contains at least 1 quarter, 1 dime and 1 nickel, which combined have a total value of 25 + 10 + 5 = 40 cents. The remaining coins must have a combined total value of 60 − 40 = 20 cents. This amount can only be made using all nickels, all dimes or a combination of nickels and dimes. So, the jar must contain only **1** quarter.

Sprint 6

Working the price backwards, we figure that Charlene's dress must have been labeled 2 × 22 = 44 dollars before the price was reduced by 50%. That $44 price was 25% = 1/4 of the original price (a reduction of 75%). So, the original price must have been 4 × 44 = **176** dollars or **176.00** dollars.

Sprint 7

The area of the rectangular region of the Norman window is 2 × 3 = 6 ft^2. The area of the semicircle is half the area of a circle with a radius of 1 foot, which is 0.5 × π × 1^2 = 0.5π ft^2. Adding these two amounts, we get 6 + 0.5π ≈ 6 + 1.57 = 7.57 ≈ **8** ft^2, to the nearest whole number.

Sprint 8

The box with dimensions 8 ft by 4 ft by 16 ft has a volume of 8 × 4 × 16 = 512 ft^3. Since 8^3 is also 512, the edge length of the cube must be **8** feet.

Sprint 9

If the first roll is 1, then there are six possible outcomes for the second roll that will not be less than 1 (1, 2, 3, 4, 5, 6). If the first roll is 2, then there are five possible outcomes for the second roll that will not be less than 2 (2, 3, 4, 5, 6). If the first roll is 3, then there are four, and so on. There are 6 × 6 = 36 possible outcomes when a die is rolled twice. Of these, 6 + 5 + 4 + 3 + 2 + 1 = 21 are such that the second number rolled is not less than the first. The desired probability is, thus, 21/36 = **7/12**.

Sprint 10

Let's start with the information from the second sentence. If 12 camels cost Sherry 96 dollars, then we know each camel costs 96 ÷ 12 = 8 dollars. This means Gary's eight camels cost him 8 × 8 = 64 dollars and the remaining 85 − 64 = 21 dollars was spent on his three pigs. Each pig, then, costs 21 ÷ 3 = 7 dollars, and the cost of two pigs is 2 × 7 = **14** dollars or **14.00** dollars.

Sprint 11

There are 4! = 24 arrangements of four different letters. Since the two Es are indistinguishable, we divide 24 by 2 to account for duplicate "words" and get **12** different "words" that can be made using all the letters of the word BEEP. Alternatively, we also could see that there are six ways to arrange the Es (EE_ _, E_E_, E_ _E, _EE_, _E_E and _ _EE). For each of these six arrangements, there are then two ways to enter the B and P, giving us a total of **12** different "words".

Sprint 12

A car averaging 50 mi/h will take 650 ÷ 50 = 13 hours to travel 650 miles. A car averaging 65 mi/h will take 650 ÷ 65 = 10 hours. Driving at the slower speed, it will take **3** hours more than driving at the faster speed.

Sprint 13

If the probability of choosing a blue marble from the bag is to be 1/2, then we must add enough blue marbles to make up half of the total number of marbles. Since there are 8 white and 6 red to start, we will need a total of 8 + 6 = 14 blue. There are 4 blue marbles to start with, so 14 − 4 = **10** blue marbles must be added to the bag.

Sprint 14

In this class of 35 students, the ratio of 6th to 7th graders is 3:4. So, 3/7 of the students are 6th graders and 4/7 are 7th graders. That's a difference of 4/7 − 3/7 = 1/7. Therefore, there are 1/7 × 35 = **5** more 7th graders than 6th graders.

Sprint 15

Based on the 4-inch difference in their heights, Carrie and Aaron could be either 60 and 64 inches tall, respectively, or 64 and 68 inches tall. But the girls are the three shortest students. So, Carrie has to be 60 inches tall, and Aaron is 64 inches tall. Since Dan is 1 inch shorter than Frank, he must be 68 inches tall, and Frank must be 69 inches tall. Betty is the shortest at 58 inches tall, so Evelyn must be 63 inches tall. The sum of Frank's and Evelyn's heights is 69 + 63 = **132** inches.

Target 1

A 5% increase means that Tia will be paying 105% of what she paid each month last year. So, the amount that Tia should expect to pay for utilities is 1.05 × 216 = **226.80** dollars.

Target 2

John travels 33 inches per second, which is 33 × 60 = 1980 inches per minute and 1980 × 60 = 118,800 inches per hour. This amounts to 118,800 ÷ 12 = 9900 feet per hour and 9900 ÷ 5280 = **1.875** mi/h.

Target 3

If the least amount any of the three pirates received is 1000 pounds, that must belong to the pirate who got two equal portions. This means that one portion must be 1000 ÷ 2 = 500 pounds. Therefore, since Long John Silver received the greatest portion of 7 equal parts, the total weight of his treasure must be 7 × 500 = **3500** pounds.

Target 4

Since the cheese will be sliced into cubes with half-inch edges, let's measure it in half inches. The 5-inch by 2-inch by 3-inch block of cheese is 10 half-inches by 4 half-inches by 6 half-inches. If we cut away all the half-inch outer layers first, we will find the inner block of cheese that does not have any wax on it. Its dimensions are 8 half-inches by 2 half-inches by 4 half-inches, so there will be 8 × 2 × 4 = **64** cubes of cheese without any wax.

Team 1

Mr. Smith's expenses are $1625 + 3 × $975 = $4550 each month. That means he has $4875 – $4550 = $325 available for entertainment. Since entertainment will account for 325/4875 = 1/15 of the budget, the sector of the graph for that category must have a central angle that is 1/15 of the 360 degrees in the full circle. That's 1/15 × 360 = **24** degrees.

Team 2

The most that a musician could earn would be 1925 dollars per week for 52 weeks, which is 1925 × 52 = 100,100 dollars. The least that a musician could earn would be $734 per week for 24 weeks, which is 734 × 24 = 17,616 dollars. The absolute difference between these extremes is 100,100 – 17,616 = **82,484** dollars or **82,484.00** dollars.

Team 3

For 2-point shots, Lisette made 67% ≈ 2/3 of 15 attempts, or 2/3 × 15 = 10 shots; Sara made 80% = 4/5 of 10 attempts, or 4/5 × 10 = 8 shots; Jen made 75% = 3/4 of 4 attempts, or 3/4 × 4 = 3 shots; and Tai made 33% ≈ 1/3 of 6 attempts, or 1/3 × 6 = 2 shots. That's a total of 10 + 8 + 3 + 2 = 23 shots made worth 2 points. For 3-point shots, Lisette made 40% = 2/5 of 5 attempts, or 2/5 × 5 = 2 shots; Sara made 50% = 1/2 of 2 attempts, or 1/2 × 2 = 1 shot; Jen did not make the 1 shot she attempted; and Tai made 50% = 1/2 of 2 attempts, or 1/2 × 2 = 1 shot. That's a total of 2 + 1 + 0 + 1 = 4 shots made worth 3 points. So, of the 45 shots attempted, 23 + 4 = 27 shots were made. The percent of shots that were successful is 27/45 = 3/5 = **60**%.

Team 4

The first card drawn determines both the suit and the rank that the next three cards must match. The probability that the next three cards match the suit is 12/51 × 11/50 × 10/49 = (12 × 11 × 10)/(51 × 50 × 49). The probability that the next three cards match the rank is 3/51 × 2/50 × 1/49 = (3 × 2 × 1)/(51 × 50 × 49). The numerator of the first probability is (12 × 11 × 10)/(3 × 2 × 1) = 220 times the numerator of the second probability, so k = **220**.

Team 5

The formula for the volume of a sphere is $V = (4/3)\pi r^3$. There are three tennis balls with radius 4 cm, so the total volume of the tennis balls is 3 × (4/3) × π × 4^3 = 4^4 × π = 256π cm^3. The formula for the volume of a cylinder is $V = \pi r^2 h$. The cylinder has a radius of 4 cm and a height of 3 × 8 cm = 24 cm. The volume of the cylinder is π × 4^2 × 24 = 384π cm^3. The empty space in the can is 384π – 256π = (384 – 256)π = **128π** cm^3.