MATHCOUNTS®

2021 Practice Competition 2

- Sprint Round 1 – 25
- Target Round 1 – 6
- Team Round 1 – 8
- Answer Key
- Solutions

PLEASE NOTE:

For this practice competition, students were given the same amount of time as they will have on the official Chapter Competition, but it included fewer Sprint, Target and Team Round problems than are on an official competition.

The Individual Score is comprised of a student's Sprint and Target scores. With fewer problems, the maximum Individual Score for this practice competition is 25 + 2 × 6 = 37 points. The maximum Individual Score on the official Chapter Competition will be 30 + 2 × 8 = 46 points.

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2021 PRACTICE COMPETITION 2 — Problems

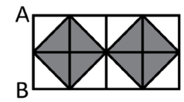
Sprint 1 Marian wishes to buy a new computer that will cost her \$300. She receives \$5 per hour for babysitting her younger brother and \$4 per hour for helping her mother with chores. Each week she babysits her brother for 4 hours and helps her mother with chores for 10 hours. How many full weeks must she work to earn enough money to buy the computer?

Sprint 2 Grady evenly distributed x candies among nine Halloween bags so that every bag received the greatest possible number of candies, but some candies were left over. What is the greatest possible number of candies that could have been left over?

Sprint 3 What is the absolute difference between the median and the mode of the data given in the stem and leaf plot shown? In this plot, 5|8 represents 58.

Tens	Units
1	2 3 4 5 5
2	2 2 2
3	1 1 8 9
4	0 1 2 3
5	2 8 9

Sprint 4 In this quilt pattern, the 16 smallest triangles are congruent, isosceles right triangles. If AB = 10 inches, what is the total area of the 8 shaded regions?



Sprint 5 If $a \# b = a^2 + b$ and $a @ b = b - a$, what is the value of $((1 \# 3) @ 2)$?

Sprint 6 What is the absolute difference between $\frac{1}{6}$ of 3 and $\frac{2}{7}$ of $2\frac{1}{3}$? Express your answer as a common fraction.

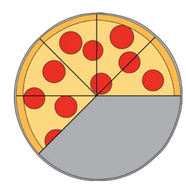
Sprint 7 What is the probability that a randomly chosen positive integer less than or equal to 24 is a factor of 24? Express your answer as a common fraction.

Sprint 8 John is twice as old as his son. In 42 years, the ratio of their ages will be 4:3. What is the son's current age?

Sprint 9 In a certain sequence of numbers, each number after the first is 3 less than twice the previous. If the third number in the sequence is 51, what is the first number of the sequence?

Sprint 10 Rita is selecting a sandwich at the deli. The deli has four types of meat, three types of cheese and two types of bread. A deluxe sandwich consists of exactly one meat type, two different types of cheese and one bread type. How many different deluxe sandwich combinations are possible?

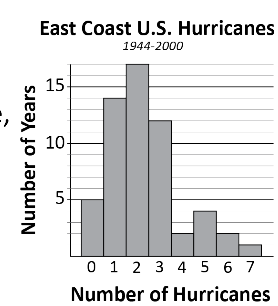
Sprint 11 Three slices have been removed from the circular pizza shown here. If the pizza originally contained eight congruent slices, what is the degree measure of the central angle of the missing sector?



Sprint 12 A group of scientists catch, tag and release 121 trout into a lake. The next day they catch 48 trout, of which 22 have been tagged. Using this ratio, how many trout would be estimated to be in the lake?

Sprint 13 Using the English alphabet, which consists of 21 consonants and 5 vowels, how many three-letter arrangements can be made in which the first letter is a consonant, the second letter is a vowel and the third letter is a consonant? Two such arrangements to include are KOM and XAX.

Sprint 14 Using data from 1944 through 2000, the histogram shows the number of years that had a particular number of hurricanes reaching the east coast of the U.S. For example, in 14 of those years, exactly one hurricane per year reached the east coast. What is the median number of hurricanes per year that reached the east coast from 1944 through 2000?



Sprint 15 The mean of seven positive integers is 16. When the smallest of these seven integers is removed, the sum of the remaining six integers is 108. What is the value of the integer that was removed?

Sprint 16 The average amount of money spent by a person who attended a local sporting event in 2000 was $8.00, of which 75% was the ticket price. In 2005, the average amount of money spent by a person who attended a local sporting event increased by 50%, but the ticket price did not increase. By how many dollars did the non-ticket costs increase from 2000 to 2005?

Sprint 17 A rectangular tile measures 3 inches by 4 inches. What is the fewest number of these tiles that are needed to completely cover a rectangular region that is 2 feet by 5 feet?

Sprint 18 Three pies and four cakes sell for $35 while four pies and five cakes sell for $44.50. What is the cost to purchase one pie and one cake?

Sprint 19 A restaurant mixes 2 gallons of milk containing 1% fat and 3 gallons of milk containing 2% fat. What is the percent of fat in the mixture? Express your answer to the nearest tenth of a percent.

Sprint 20 It would take John 6 hours to paint a particular room by himself. It would take Tom 12 hours to paint the same room by himself. If John and Tom work together, each at his individual rate, how many hours will it take them to paint the room?

Sprint 21 When Sarah rowed down Black River with the current, she took 1 hour to go 4 miles. When she rowed back the same distance, at the same rowing speed, but against the current, her trip required 2 hours. What was the speed, in miles per hour, of the current in Black River?

Sprint 22 It cost Mr. Andrews $200,000 to build a house. He sold it to Ms. Bond at a 10% profit. Later Ms. Bond sold it to Mr. Cash at a 10% loss. What is the absolute difference between the amount Ms. Bond bought the house for and the amount Ms. Bond sold the house for?

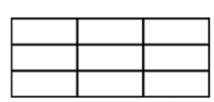
Sprint 23 What is the units digit of the integer value of $2007^{2008} + 2008^{2007}$?

Sprint 24 A six-sided die has faces labeled 2, 3, 5, 7, 11 and 13. If this die is rolled twice, what is the probability that the product of the two numbers rolled will be even? Express your answer as a common fraction.

Sprint 25 A cube of edge length 3 units has each face painted orange. The cube is then cut into 27 unit cubes. How many of these unit cubes have exactly two faces painted orange?

Target 1 If $f(x) = \frac{(3x-2)}{(x-2)}$, what is the value of $f(-2) + f(-1) + f(0)$? Express your answer as a common fraction.

Target 2 How many rectangles are in the array shown?

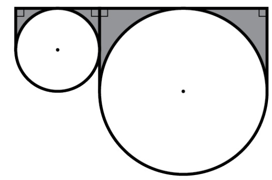


Target 3 A 1/2-mile long train enters a 2-mile long tunnel traveling at a speed of 10 mi/h. How many minutes pass from the time the front of the first train car enters the tunnel until the rear of the last train car exits the tunnel?

Target 4 Five pirates share the treasure with Long John Silver. If the treasure is split, by weight, in the ratio 2:5:7:10:20:50, and the least amount any of the six pirates receives is 14,000 pounds of gold, what is the total weight of the treasure?

Target 5 What is the greatest whole number that MUST be a factor of the sum of any four consecutive positive odd numbers?

Target 6 In the figure shown, the smaller circle has a radius of 2 feet and the larger circle has a radius of 4 feet. What is the total area of the four shaded regions? Express your answer as a decimal to the nearest hundredth.



Team 1 What is the maximum number of cubes of edge length 2 inches that will fit inside a rectangular box whose interior measures 1 foot by 14 inches by 16 inches?

Team 2 The decimal point of a positive real number is moved two digits to the right to form a different real number. When the absolute difference between these two real numbers is divided by 11, the answer is 21. What is the original real number? Express your answer as a common fraction.

Team 3 A silent auction was held at Little M.S. and some of the data are given in the table shown. Some of the digits of the bids were smudged, as indicated by the underlined asterisks. The amount of each successive bid is greater than the previous bids for that item, and all bids are whole numbers of dollars. The last (and highest) bidder wins the item for the price bid. What is the least combined amount that could have been paid for these four items?

Necklace	Las Vegas Trip	Ski Passes	Autographed Football
Mike $12	Lou $100	Mike $30	Jill $15
Angie $<u>*</u><u>*</u>	Jill $<u>*</u>10	Pat $36	Mike $<u>*</u>5
	Jack $<u>*</u>10	Andrew $3<u>*</u>	

Team 4 A standard deck of cards includes four suits, each of which contains an ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, jack, queen and king. Four cards are drawn at random without replacement from a standard deck of 52 cards. What is the probability that all four aces are drawn? Express your answer as a common fraction.

Team 5 How many integers are solutions to the equation $(x - 2)^{(25 - x^2)} = 1$?

Team 6 If Ella rolls a standard six-sided die until she rolls the same number on consecutive rolls, what is the probability that her 10th roll is her last roll? Express your answer as a decimal to the nearest thousandth.

Team 7 Three friends shared a full bag of jellybeans. Mike took 1/3 of the jellybeans in the full bag, Zac took 1/2 of the jellybeans in the full bag and Kary took what was left. Mike ate 1/2 of his jellybeans, Zac ate 1/3 of his jellybeans and Kary ate all of hers. If Mike and Zac were left with a combined total of 45 jellybeans, how many jellybeans did Kary eat?

Team 8 How many distinct five-digit positive integers can be formed using the digits 2, 3, 4, 7 and 8 if only the digit 2 can be used more than once?

2021 Practice Competition 2 — Answers

Sprint 1	5	**Target 1**	14/3	**Team 1**	336
Sprint 2	8	**Target 2**	36	**Team 2**	7/3
Sprint 3	9	**Target 3**	15	**Team 3**	285 or 285.00
Sprint 4	100	**Target 4**	658,000	**Team 4**	1/270,725
Sprint 5	–2	**Target 5**	8	**Team 5**	4
Sprint 6	1/6	**Target 6**	8.58	**Team 6**	0.039
Sprint 7	1/3			**Team 7**	15
Sprint 8	21			**Team 8**	501
Sprint 9	15				
Sprint 10	24				
Sprint 11	135				
Sprint 12	264				
Sprint 13	2205				
Sprint 14	2				
Sprint 15	4				
Sprint 16	4 or 4.00				
Sprint 17	120				
Sprint 18	9.50				
Sprint 19	1.6				
Sprint 20	4				
Sprint 21	1				
Sprint 22	22,000 or 22,000.00				
Sprint 23	3				
Sprint 24	11/36				
Sprint 25	12				

Sprint 1

In one week, Marian will make \$5 × 4 = \$20 for babysitting her brother and \$4 × 10 = \$40 for helping her mother with chores. So, Marian makes \$20 + \$40 = \$60 per week. Thus, she will need to work 300/60 = **5** full weeks to earn enough to buy the new computer.

Sprint 2

The greatest possible remainder when dividing by nine is 8. There could be **8** candies left over. If there were nine or more candies left over, another candy would have been put into each bag.

Sprint 3

We see that there are 19 values represented in the stem and leaf plot in increasing order (from left to right, top to bottom). Therefore, the median is the 10th value, which is 31. The mode is 22, of which there are three occurrences. So, the absolute difference of the median and mode of the data is 31 – 22 = **9**.

Sprint 4

Given that the 16 smallest triangles are congruent, isosceles right triangles, we know that the entire quilt pattern is a rectangle. The left side is 10 inches, and we can determine that the top side is then 20 inches. The area of the entire pattern is 10 × 20 = 200 in^2. Since eight of the 16 smallest triangles are shaded, we know that 1/2 of the total area is shaded, which is 1/2 × 200 = **100** in^2.

Sprint 5

Evaluating ((1 # 3) @ 2) according to the rules for the operations # and @, we get $(2 - (1^2 + 3)) = 2 - 4 = \mathbf{-2}$.

Sprint 6

One-sixth of 3 is 1/6 × 3/1 = 3/6 = 1/2. Two-sevenths of $2\frac{1}{3}$ is 2/7 × 7/3 = 2/3. The absolute difference is |1/2 – 2/3| = |3/6 – 4/6| = **1/6**.

Sprint 7

First, we need to know the factors of 24. They are 1, 2, 3, 4, 6, 8, 12 and 24. There are a total of 24 positive integers less than or equal to 24, and 8 of those positive integers are factors of 24. So, the probability that a randomly chosen positive integer less than or equal to 24 is a factor of 24 is 8/24 = **1/3**.

Sprint 8

Let J be John's age and S be his son's age. We have $J = 2S$ right now and $(J + 42)/(S + 42) = 4/3$ in 42 years. The cross product of the second equation is $3(J + 42) = 4(S + 42)$, or $3J + 126 = 4S + 168$. Substituting $2S$ for J and solving, we get $3(2S) + 126 = 4S + 168$. So, $2S = 42$ and $S = 21$. John's son must be **21** years old.

Sprint 9

If we let n represent the first term of the sequence, then the second term has value $2n - 3$, and the third term has value $2(2n - 3) - 3 = 4n - 9$. Since we are told that the third term is 51, it follows that $4n - 9 = 51$. So, $4n = 60$ and $n = \mathbf{15}$.

Sprint 10

There are four different options for the meat. There are three different ways to choose two of the three different cheeses for the deluxe sandwich. (You also can think of this as three options for which cheese to leave off the sandwich.) Finally, there are two bread options. By the Fundamental Counting Principle, there must be $4 \times 3 \times 2 =$ **24** combinations.

Sprint 11

A complete circle contains 360 degrees in its central angle. If the pizza had 8 congruent slices, the central angle of each slice was $360/8 = 45$ degrees. The three slices that were removed, and therefore the missing sector of the pizza, would make up $45 \times 3 =$ **135** degrees.

Sprint 12

The idea is that the ratio of the 121 trout that were tagged and released on the first day compared to the unknown number of trout in the lake t is proportional to the ratio of the tagged fish caught to the total fish caught on the second day, which is $22/48 = 11/24$. We set up the proportion $121/t = 11/24$, and solve for t. The cross product is $11t = 121 \times 24$, so $t = (121 \times 24)/11 = 11 \times 24 = 264$. We would estimate that there are **264** trout in the lake.

Sprint 13

There are 21 consonants that could be the first letter of the three-letter arrangement, 5 vowels that could be the second letter and the same 21 consonants that could be the third letter. Thus, there are $21 \times 5 \times 21 =$ **2205** possible three-letter arrangements.

Sprint 14

From 1944 through 2000, inclusive, there are 57 years, and therefore 57 values represented in the histogram in increasing order. The median is the 29th value. So, the 5 years with 0 hurricanes and the 14 years with 1 hurricane per year account for the first $5 + 14 = 19$ values of the data set. There are 17 years with 2 hurricanes per year, so the 20th through the 37th values in the data set are all 2. Therefore, the 29th value, the median number of hurricanes per year, is **2** hurricanes.

Sprint 15

Let s be the sum of the seven positive integers whose mean is 16. We can write the equation $s \div 7 = 16$, so $s = 16 \times 7 = 112$. If the sum of the integers is 108 when the smallest integer is removed, then the integer that was removed must be $112 - 108 =$ **4**.

Sprint 16

The average amount of money spent by a person at a sporting event in 2000 was \$8.00. The ticket price was \$8.00 × 0.75 = \$6.00. In 2005, the amount spent increased by \$8.00 × 0.50 = \$4.00 to a total of \$8.00 + \$4.00 = \$12.00. However, the ticket price of \$6.00 did not change. Therefore, the non-ticket costs increased from \$8.00 – \$6.00 = \$2.00 to \$12.00 – \$6.00 = \$6.00. That's an increase of \$6.00 – \$2.00 = \$**4** or \$**4.00**.

Sprint 17

It takes six 4-inch lengths to make 2 feet, and it takes twenty 3-inch lengths to make 5 feet. So, we have 6 of these tiles going down and 20 of these tiles going across: $6 \times 20 =$ **120** tiles. Alternatively, it takes eight 3-inch lengths to make 2 feet and fifteen 4-inch lengths to make 5 feet. That's 8 tiles going down and 15 tiles going across: $8 \times 15 =$ **120** tiles.

Sprint 18

Let *c* represent the cost of one cake, and let *p* represent the cost of one pie. From the information given, we can write the equations $3p + 4c = 35$ and $4p + 5c = 44.50$. Subtracting the first equation from the second equation gives $(4p + 5c) - (3p + 4c) = 44.50 - 35$, so $p + c = 9.50$. Therefore, the cost of one pie and one cake is $**9.50**.

Sprint 19

The 2 gallons of milk containing 1% fat contribute 2 × 0.01 = 0.02 gallon of fat, and the 3 gallons of milk containing 2% fat contribute 3 × 0.02 = 0.06 gallon of fat. Thus, there is 0.02 + 0.06 = 0.08 gallon of fat in the 2 + 3 = 5-gallon mixture of milk. Thus, the percent of fat in the mixture is 0.08 ÷ 5 = 0.016 = **1.6**%.

Sprint 20

Since Tom's rate of painting is exactly double John's rate, John will paint exactly double what Tom will paint in the same amount of time. So, John will paint 2/3 of the room, while Tom will paint 1/3 of the room, each working at their own individual rates. Thus, Tom and John will take (2/3) × 6 = (1/3) × 12 = **4** hours to paint the room.

Sprint 21

Let *b* represent Sarah's speed in still water, and let *c* represent the speed of the water's current. We know that Sarah traveled 4 miles going downstream in 1 hour, with the current. Using the formula *distance = rate × time*, we can write an equation to represent her trip downstream: $4 = (b + c) \times 1$. When traveling upstream, against the current, Sarah went the same distance but took 2 hours, which can be represented by the equation $4 = (b - c) \times 2$. This equation simplifies to $b - c = 2$. Subtracting the upstream equation from the downstream equation and solving for *c* gives us $(b - c) - (b + c) = (2 - 4)$, so $-2c = -2$ and $c =$ **1** mi/h.

Sprint 22

In order for Mr. Andrews to make a 10% profit in selling the house, he must have sold it to Ms. Bond for (1 + 0.1) × 200,000 = $220,000. Then, in order for Ms. Bond to lose 10%, she must have sold it to Mr. Cash for (1 – 0.1) × 220,000 = 198,000. Therefore, the absolute difference between the amount Ms. Bond bought the house for and the amount she sold the house for is 220,000 – 198,000 = $**22,000** or $**22,000.00**.

Sprint 23

Only the units digit of 2007 to a power will contribute to the units digit of the final sum. If we consider the first few powers of 7, we notice that there is a repeating 4-cycle in the units digit: $7^1 = 7$, $7^2 = 49$, $7^3 = 343$, $7^4 = 2401$, $7^5 = 16{,}807$, etc. The pattern is 7, 9, 3, 1, 7, 9, 3, 1, etc. The 2008th power of 7 will have a units digit of 1, since 2008 is a multiple of 4. Similarly, the units digit of the powers of 2008 will depend only on the units digits of the powers of 8. The 4-cycle pattern in the units digit for powers of 8 is 8, 4, 2, 6, 8, 4, 2, 6, etc. The 2007th power of 8 will have a units digit equal to the third number in the cycle, which is a 2. The units digit of the sum $2007^{2008} + 2008^{2007}$ is thus 1 + 2 = **3**.

Sprint 24

It is easier to calculate the probability that the product of the numbers rolled will not be even, then subtract this value from 1. The primes on the die are 2, 3, 5, 7, 11 and 13. The product of the two numbers rolled will be odd as long as the 2 is not rolled. There is a 5/6 chance of rolling one of the other primes each roll, so the probability is 5/6 × 5/6 = 25/36 that the product will be odd. Subtracting 25/36 from 36/36, we find that the probability is **11/36** that the product will be even.

Sprint 25

At each of the eight corners of the original cube is a unit cube that has exactly three faces painted orange. In the center of each of the six faces of the original cube is a unit cube that has exactly one face painted orange. In the center of the original cube, there is a unit cube that has no faces painted orange. On each of the 12 edges of the original cube, there is a unit cube that has exactly two faces painted orange. Thus, the number of unit cubes with exactly two faces painted orange is **12** unit cubes.

Target 1

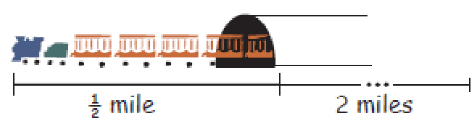
Evaluating *f*(–2) gives (3 × (–2) – 2)/(–2 – 2) = –8/(–4) = 2. Evaluating *f*(–1) gives (3 × (–1) – 2)/(–1 – 2) = –5/(–3) = 5/3. Evaluating *f*(0) gives (3 × 0 – 2)/(0 – 2) = –2/(–2) = 1. Therefore, *f*(–2) + *f*(–1) + *f*(0) = 2 + 5/3 + 1 = 6/3 + 5/3 + 3/3 = **14/3**.

Target 2

There are 9 rectangles made up of a single small rectangle each. There are 12 rectangles composed of two small rectangles each. There are 6 rectangles composed of three small rectangles each. There are 4 rectangles composed of four small rectangles each. There are also 4 rectangles composed of six small rectangles each. Finally, there is 1 rectangle composed of all nine small rectangles. Therefore, there are 9 + 12 + 6 + 4 + 4 + 1 = **36** rectangles in the array.

Target 3

This problem is a little tricky because we have to figure out how far the train has traveled from the time the front of the first train car entered the tunnel to the time the rear of the last train car exited the tunnel. The tunnel is 2 miles long, but the train has actually traveled farther than this. The front of the train entered the tunnel, traveled 2 miles, exited the tunnel, and then traveled another half-mile before the rear of the last train car was completely out of the tunnel. So, we need to find the number of minutes it took for this train to travel 2 + 0.5 = 2.5 miles. We can use the equation *distance = rate × time*, where the distance is 2.5 miles, the rate is the speed of the train 10 mi/h, and the time *t* hours is how long it took the train to travel 2.5 miles going 10 mi/h. Doing so, we have 2.5 = 10*t*, so *t* = 2.5 ÷ 10 = 0.25 hours. The problem asked for the number of minutes, and 0.25 hours is a quarter of an hour, which is **15** minutes.



Target 4

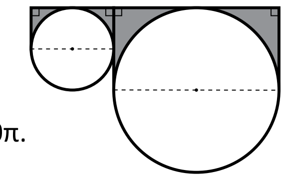
If the pirates split the treasure, by weight, in the ratio 2:5:7:10:20:50, then there are a total of 2 + 5 + 7 + 10 + 20 + 50 = 94 equal portions of treasure. If the least amount any of the six pirates received is 14,000 pounds, that must belong to the pirate who received two equal portions. This means that one portion must be 14,000 ÷ 2 = 7000 pounds. The total weight of the treasure must be 94 × 7000 = **658,000** pounds.

Target 5

Let *x* be the least of the four odd numbers. Then the sum of the four consecutive positive odd numbers is *x* + *x* + 2 + *x* + 4 + *x* + 6 = 4*x* + 12 = 4(x + 3). We know *x* is an odd number. Since the sum of two odd numbers is even, it follows that *x* + 3 must be even, meaning *x* + 3 = 2*n* for some quantity *n*. Rewriting the sum, we have 4(*x* + 3) = 4(2*n*) = 8*n*. So, the greatest whole number that must be a factor of this sum is **8**.

Target 6

First, we can create rectangles by drawing the diameter of each circle that intersects its respective tangent lines, as shown. Each rectangle has a length equal to that circle's radius and a width equal to its diameter. The area of the smaller rectangle is $4 \times 2 = 8$ ft^2. The area of the larger rectangle is $8 \times 4 = 32$ ft^2. Their total area is $32 + 8 = 40$ ft^2. These areas include the shaded areas and the unshaded semicircles. So, subtracting the areas of the semicircles will give us the areas of the shaded regions. The area of the smaller semicircle, which has radius 2 ft, is $1/2 \times \pi \times r^2 = 1/2 \times \pi \times 2^2 = 2\pi$ ft^2. The area of the larger semicircle, which has radius 4 ft, is $1/2 \times \pi \times r^2 = 1/2 \times \pi \times 4^2 = 8\pi$ ft^2. So, the total area to be subtracted is $2\pi + 8\pi = 10\pi$. The total area of the shaded regions, then, is $40 - 10\pi \approx$ **8.58** ft^2.



Team 1

Assume the interior of the box is 1 foot = 12 inches tall, 14 inches wide and 16 inches deep. We could stack the 2-inch cubes $12 \div 2 = 6$ high and $14 \div 2 = 7$ wide, creating a layer of $6 \times 7 = 42$ cubes. With an interior depth of 16 inches, we could fill the box with $16 \div 2 = 8$ such layers of 42 cubes, for a total of $8 \times 42 =$ **336** cubes.

Team 2

Multiplying a number by 100 has the effect of moving the decimal point two digits to the right. So, if the original number is x, then the new number that is formed by moving the decimal point two digits to the right is $100x$. Since x is a positive real number, we know that $100x > x$. So, the absolute difference of these two values is $100x - x = 99x$. We are told that $99x/11 = 21$, so $9x = 21$ and $x = 21/9 =$ **7/3**.

Team 3

Since we know Angie's bid for the necklace had to be more than \$12 and a whole number of dollars, the least amount that her bid could have been was \$13. For the Las Vegas trip, the least amount Jill could have bid was \$110, which means the least amount Jack could have bid was \$210. For the ski passes, the least amount Andrew could have bid was \$37, and for the autographed football, the least amount Mike could have bid was \$25. Based on the rules of the silent auction, Angie, Jack, Andrew and Mike were the winners, which means the least combined amount that could have been paid for the four items is \$13 + \$210 + \$37 + \$25 = \$**285** or \$**285.00**.

Team 4

The probability that one of the four aces is drawn first is 4/52. The probability that one of the three remaining aces is drawn next is 3/51. The probability that one of the two remaining aces is drawn next is 2/50. Finally, on the fourth draw, the probability that the last ace is drawn is 1/49. So, the probability that the four randomly drawn cards are the four aces is $4/52 \times 3/51 \times 2/50 \times 1/49 =$ **1/270,725**.

Team 5

There are two cases in which a power of $(x - 2)$ is equal to 1: either $x - 2 = 1$, or $x - 2 = -1$ and $25 - x^2$ is even. Then there is the case in which the exponent is $25 - x^2 = 0$. Let's examine all three cases.

Case 1: If $x - 2 = 1$, then $x = 3$. So, we have $(x - 2)^{25 - x^2} = (3 - 2)^{25 - 3^2} = 1^{16} = 1$. Thus, $x = 3$ is a solution.

Case 2: If $x - 2 = -1$, then $x = 1$. So, we have $(x - 2)^{25 - x^2} = (1 - 2)^{25 - 1^2} = (-1)^{24} = 1$. Thus, $x = 1$ is a solution.

Case 3: If $25 - x^2 = 0$, then $x^2 = 25$ and $x = \pm 5$. So, we have $(x - 2)^{25 - x^2} = (5 - 2)^{25 - 5^2} = 3^0 = 1$ and $((-5) - 2)^{25 - (-5)^2} = (-7)^0 = 1$. That means both $x = 5$ and $x = -5$ are solutions.

That gives us a total of **4** integers that are solutions.

Team 6

For her first two rolls to be different, Ella's first roll can be any of 6 values and her second roll can be any of 5. So, the probability is (6 × 5)/(6 × 6) = 30/36 = 5/6. The probabilities that the third roll is different from the second, the fourth roll is different from the third, and so forth until the ninth roll is different from the eighth are each 5/6 as well. The probability that the tenth roll is the same as the ninth roll is 1/6. Thus, the probability that her tenth roll is her last roll is $(5/6)^8 \times (1/6) \approx$ **0.039**.

Team 7

Mike took 1/3 of the jellybeans in the bag and then ate 1/2 of those, so he actually had 1/6 of the entire original bag of jellybeans left. Zac took 1/2 of the jellybeans in the bag and then ate 1/3 of those, leaving 1/2 – (1/3 × 1/2) = 1/2 – 1/6 = 1/3 of the entire original bag of jellybeans left. Between the two of them, they now have 45 jellybeans, which must be 1/6 + 1/3 = 1/6 + 2/6 = 3/6 = 1/2 of the entire original bag of jellybeans. Thus, the total number of jellybeans that were originally in the bag was 45 × 2 = 90 jellybeans. Between Mike and Zac, they originally took 1/3 + 1/2 = 2/6 + 3/6 = 5/6 of the entire original bag, leaving just 1/6 of the bag for Kary. So, Kary ate (1/6) × 90 = **15** jellybeans.

Team 8

The number of five-digit integers where none of the digits are repeated is 5! = <u>120</u>. The number of different arrangements for a 5-digit integer containing two 2s (i.e. 22_ _ _, 2_2_ _, etc.) is 5!/2!3! = (5 × 4)/2 = 10 arrangements. For each arrangement, we have to fill in the other three digits. There are 4 choices for the first digit, 3 choices for the second digit and 2 choices for the last digit. That's 4 × 3 × 2 = 24 combinations of digits for each arrangement, giving us a total of 24 × 10 = <u>240</u> such 5-digit integers containing two 2s. The number of different arrangements for a 5-digit integer containing three 2s is 5!/3!2! = (5 × 4)/2 = 10 arrangements. For the remaining two digits, we have 4 choices for the first digit and 3 choices for the second digit. That's 4 × 3 = 12 combinations of digits for each arrangement, giving us a total of 12 × 10 = <u>120</u> such 5-digit integers containing three 2s. The number of different arrangements for a 5-digit integer containing four 2s is 5!/4!1! = 5 arrangements. For each arrangement, there are 4 choices for the digit that is not a 2, giving us a total of 5 × 4 = <u>20</u> such 5-digit integers containing four 2s. Finally, there is only <u>1</u> arrangement for the 5-digit number containing five 2s. So, in total, there are 120 + 240 + 120 + 20 + 1 = **501** such integers.