MATHCOUNTS®

2021 Practice Competition 3

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2021 Practice Competition 3 — Problems

Sprint 1 How many integers between 500 and 1000 contain both the digits 3 and 4?

Sprint 2 Tom rides the bus part of the way to school and then he walks the rest of the way. He walks five minutes longer than he rides. The whole trip takes 27 minutes. For how many minutes does he walk?

Sprint 3 Liberty Middle School's enrollment increased to 660 students. This is an increase of 10% over last year's enrollment. What was last year's enrollment?

Sprint 4 The distance from the earth to the sun is 93,000,000 miles, and light travels at 186,000 miles per second. How many seconds does light from the sun take to reach the earth?

Sprint 5 Each of four test scores in Connie's class is to be weighted equally. On the first three tests, Connie scored 80%, 90% and 95%. What percent must she score on her fourth test to have an overall average of exactly 90%?

Sprint 6 Haley has enlarged a 3-inch by 5-inch picture so that both the length and width are tripled. The area of the enlarged photo is how many times the area of the original photo?

Sprint 7 What is the least positive integer divisible by each of 1, 3, 5 and 7?

Sprint 8 If $m \lozenge n = (m^2 - n) \div n$ for all real numbers m and n, where $n \neq 0$, what is the value of $6 \lozenge 3$?

Sprint 9 New York and Denver are in different time zones. When it is noon in New York, it is 10:00 a.m. in Denver. At 2:00 p.m. in New York, a train departs, and it arrives in Denver 45 hours later. What time is it in Denver when the train arrives?

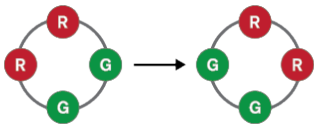
Sprint 10 A month ago, the ratio of nurses to doctors on a hospital staff was 3:5. Since that time, two additional nurses joined the staff, no nurses left and the number of doctors remained the same. The ratio of nurses to doctors on the hospital staff is now 4:5. How many nurses are **now** on the staff?

Sprint 11 A sequence is formed by multiplying each term by 3 and then adding 3 to get the next term. If the third term is 39, what is the value of the first term?

Sprint 12 Bill and Jill both exercise on Monday, January 1. Bill exercises every 5th day, and Jill exercises every 4th day. What is the next date on which Bill and Jill both exercise?

Sprint 13 Consider the square with vertices at (3, 3), (−3, 3), (−3, −3) and (3, −3). How many points with integer coordinates lie strictly in the interior of this square?

Sprint 14 A bracelet is made by stringing together four beads. Each bead is either red or green. How many different color patterns are possible for the bracelet, where patterns are considered the same if rotating one will produce the other, as shown here?



Sprint 15 To create a unique house paint color, Melton mixes together a sample that is 12 gallons of red, 2.5 gallons of yellow and 0.5 gallons of blue paint. He then mixes a main batch of paint using 30 gallons of yellow paint and enough red and blue paint to maintain the original ratio. How many total gallons of paint did he use to make the main batch of paint?

Sprint 16 How many different squares can be formed by using four of the evenly spaced dots shown as vertices of the square?

2021 Practice Competition 3 — Problems

Sprint 17 Xinran walks 3 mi/h uphill, 4 mi/h on flat land and 5 mi/h downhill. If he walks one mile uphill, then one mile on flat land and then returns by the same route to his starting point, how many minutes does he walk?

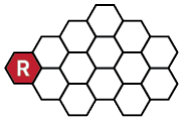
Sprint 18 Sassy Fashions buys dresses at wholesale and then marks them up for retail sale. They recently sold a dress at a 40% discount off their marked-up price. What percent mark-up did they originally apply to the dress if they broke even on the sale? Express your answer to the nearest whole percent.

Sprint 19 For integers a, b and k, we know that $a > 12$, $b < 20$ and $a < b$. If $b = 7k$, what is the value of k?

Sprint 20 If one quart of paint is exactly enough for two coats of paint on a 9-foot by 10-foot wall, how many quarts of paint are needed to apply one coat of paint to a 10-foot by 12-foot wall? Express your answer as a common fraction.

Sprint 21 A mixture is made with 45 ounces of a 10% saline solution and x ounces of a 70% saline solution. The resulting mixture is a 25% saline solution. What is the value of x?

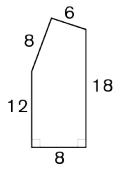
Sprint 22 In the figure, the hexagon with the "R" is colored red, and each of the other hexagons will be colored red, yellow or green, so that no two hexagons with a common side are the same color. In how many different ways can the figure be colored?



Sprint 23 Five numbered balls, each with a different number from 1 through 5, are placed in a bowl. If Josh randomly chooses two balls, with replacement, what is the probability that the two numbers on the selected balls have a product that is even and greater than 10? Express your answer as a common fraction.

Sprint 24 On Claudia's birthday in 2019, her age was four times her brother's age on that day. On her birthday in 2020, her age was three times her brother's age on that day. In what year will Claudia's age, on her birthday, be twice her brother's age on that day?

Sprint 25 What is the area of the pentagon shown, with the indicated side lengths in inches?

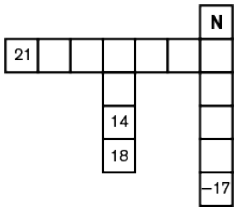


Sprint 26 A bag contains exactly three red marbles, five yellow marbles and two blue marbles. If three marbles are drawn from the bag without replacement, what is the probability that all three will be the same color? Express your answer as a common fraction.

Sprint 27 A peep increased by 25% is a pop. A pop decreased by 40% is a slug, and a slug increased by 100% is a slap. What percent of a peep is a slap?

Sprint 28 If Jonah reverses the two digits of his age, divides the resulting number by three, and then adds 20, the result is Jonah's age. How old is Jonah?

Sprint 29 In the figure shown, the sequence of integers in the row of squares and in each of the two columns of squares form three distinct arithmetic sequences. What is the value of N?



2021 Practice Competition 3 — Problems

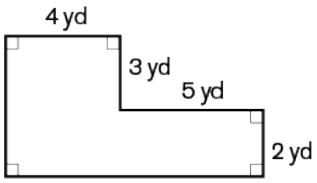
Sprint 30 If $\frac{a}{b} = \frac{3}{4}$, $\frac{b}{c} = \frac{8}{9}$ and $\frac{c}{d} = \frac{2}{3}$, what is the value of $\frac{ad}{b^2}$? Express your answer as a common fraction.

Target 1 Helga invested $1000 at 5% interest, compounded annually. What is the total amount of interest that Helga will have earned by the end of the fourth year? Express your answer to the nearest dollar.

Target 2 What is the absolute difference between the sum of the first 10 positive multiples of 5 and the sum of the first 10 positive, even integers?

Target 3 The Moisture Co. produces disinfecting wipes. If 70 wipes completely fill a rectangular carton measuring 6 inches by 4 inches by 2 inches, and 100 wipes completely fill a rectangular carton measuring 6 inches by 4 inches by *h* inches, what is the value of *h*? Express your answer as a decimal to the nearest tenth.

Target 4 Carpet costs $21.95 per square yard and the padding to put under it costs $2.55 per square yard. Felix plans to install padding and carpet in the region shown in the figure. What is the total cost of the carpet and padding needed to exactly cover the room?



Target 5 In how many different ways can four students stand in a straight line if two of the students refuse to stand next to each other?

Target 6 Given that $-3 \le x \le 2$ and $20x^2 = y - 24$, what is the least possible value for *y*?

Target 7 Bill, Phil and Jenny are siblings. Bill is twice as old as Phil. Jenny is two years younger than Bill. Currently, their dad is twice as old as the sum of their ages. In nine years, their dad's age will be equal to the sum of his three kids' ages at that time. What is Jenny's current age?

Target 8 An ammonia and water mixture fills a five-gallon container. Eighty percent of the mixture is ammonia, but some of the mixture will be drained and replaced with pure water. If a five-gallon mixture of fifty percent ammonia is desired, how many quarts of the mixture need to be drained before the water is added, given that 4 quarts equals a gallon? Express your answer as a decimal to the nearest tenth.

Team 1 A competition problem requires one hour to fully develop (write, proofread, edit and typeset). This problem is then given to 30,000 students, each working an average of 24 seconds to solve the problem. What is the ratio of a problem's development time to the total time spent by the students to solve the problem? Express your answer as a common fraction.

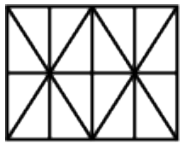
Team 2 One interior angle of a convex polygon measures 160 degrees. The rest of the interior angles of the polygon each measure 112 degrees. How many sides does the polygon have?

Team 3 An octopus has 8 tentacles and 1 head. A jellyfish has 20 tentacles and no head. A cow has 4 legs and 1 head. Farmer Brown, who only raises octopi, jellyfish and cows on her farm, has animals with a total of 17 heads, 196 tentacles and 20 legs. How many animals does she have?

Team 4 Keisha has 45 coins in her piggy bank, consisting of only nickels, dimes and quarters. She has one fewer nickel than she has dimes, and one fewer quarter than three times her number of nickels. How many nickels does she have?

Team 5 Joann rode her bike at an average speed of 12 mi/h for three and a half hours. If Frank rides for 3 hours, at what average speed would he have to ride his bike to travel the same distance as Joann?

Team 6 How many triangles of any size are in the figure shown?



Team 7 The positive integer divisors of 175, except 1, are arranged around a circle so that every pair of adjacent integers has a common factor greater than 1. What is the sum of the two integers adjacent to 7?

Team 8 What is the 200th term of the increasing sequence of positive integers formed by omitting only the perfect squares?

Team 9 A bag contains 3 red balls, 4 green balls and 5 yellow balls. If balls are drawn one at a time without replacement, what is the probability that the first yellow ball is drawn on the eighth draw? Express your answer as a common fraction.

Team 10 In Heidi's history class, the only grades that count toward the semester average are the 6 tests she has already taken and the upcoming final exam. The final exam counts as two tests. Heidi has determined that if she earns 99 points on the final, she will have exactly a 90-point average for the semester. On average, how many points did Heidi score on each test prior to the final exam?

2021 Practice Competition 3 Answers

Sprint 1	10
Sprint 2	16
Sprint 3	600
Sprint 4	500
Sprint 5	95
Sprint 6	9
Sprint 7	105
Sprint 8	11
Sprint 9	9 or 9:00 or 09:00
Sprint 10	8
Sprint 11	3
Sprint 12	01 (Jan) 21
Sprint 13	25
Sprint 14	6
Sprint 15	180
Sprint 16	10
Sprint 17	62
Sprint 18	67
Sprint 19	2
Sprint 20	2/3
Sprint 21	15
Sprint 22	2
Sprint 23	1/5
Sprint 24	2023
Sprint 25	144
Sprint 26	11/120
Sprint 27	150
Sprint 28	48
Sprint 29	−7
Sprint 30	81/64

Target 1	216 or 216.00
Target 2	165
Target 3	2.9
Target 4	735 or 735.00
Target 5	12
Target 6	24
Target 7	6
Target 8	7.5

Team 1	1/200
Team 2	6
Team 3	22
Team 4	9
Team 5	14
Team 6	36
Team 7	210
Team 8	214
Team 9	1/792
Team 10	87

2021 Practice Competition 3 Solutions

Sprint 1

From 500 to 600, we have 2 numbers, 534 and 543. Similarly, for 601–700, 701–800, 801–900 and 901–1000, there are 2 numbers containing 3 and 4 in each 100-number range. So, 2 × 5 = **10** integers contain both the digits 3 and 4.

Sprint 2

Let B be the number of minutes Tom spends on the bus, and let W be the number of minutes Tom spends walking. Since Tom walks five minutes longer than he rides the bus, we have $B = W - 5$. The time for the whole trip can be expressed as $27 = B + W = (W - 5) + W = 2W - 5$. Solving $2W - 5 = 27$ for W, we get $2W = 32$, so $W = 16$. That means Tom must have walked for **16** minutes.

Sprint 3

Given that 660 students is an increase of 10% over last year's enrollment, we can say that $660 = 1.10x$, where x = last year's enrollment. Solving this equation, we get $x = 660 \div 1.10 = 600$. So, last year's enrollment was **600** students.

Sprint 4

We need to divide the 93,000,000 miles from the sun to the earth by the 186,000 miles per second that light travels to find out how many seconds the light takes to reach the earth. If we make a fraction, we get 93,000,000/186,000, which reduces easily to 93,000/186. Let's think about this as (93/186) × 1000, which reduces to (1/2) × 1000. We can see that it takes **500** seconds for the light from the sun to reach the earth.

Sprint 5

If Connie is to have an overall average of 90% on four tests, she needs a total of 4 × 90 = 360 percentage points. So far, she has 80 + 90 + 95 = 265 points. So, she must score exactly 360 − 265 = **95**% on the fourth test.

Sprint 6

Haley enlarged each of the dimensions of the picture by a factor of 3, so the new picture has dimensions of 3 × 3 = 9 inches by 3 × 5 = 15 inches. The area of the enlarged photo is then 9 × 15 = 135 square inches. The area of the original picture was 3 × 5 = 15 square inches. Dividing 135/15 gives that the enlarged photo is **9** times the area of the smaller photo.

Sprint 7

Because each of 1, 3, 5 and 7 is a prime number, we know the least positive integer divisible by all these numbers is 1 × 3 × 5 × 7 = **105**.

Sprint 8

Using the given rule for the operation ◊, we can calculate $6 \lozenge 3$ as $(6^2 - 3) \div 3 = (36 - 3) \div 3 = 33 \div 3 = \mathbf{11}$.

Sprint 9

If the train departs at 2:00 p.m. in New York and arrives in Denver 45 hours later, this is equivalent to saying that the train leaves New York at noon, Denver time, and arrives in Denver 45 hours later. If it were to arrive in Denver 48 hours later (instead of 45) it would be exactly two days later or noon in Denver time. But since it takes only 45 hours instead of 48, it must arrive three hours earlier, at **9:00** a.m.

2021 Practice Competition 3 Solutions

Sprint 10

Let n and d represent the numbers of nurses and doctors, respectively. Based on the given information, for last month, we have the proportion $n/d = 3/5$. Cross-multiplying and solving for d, we get $3d = 5n$, so $d = (5/3)n$. For this month, we have the proportion $(n + 2)/d = 4/5$. Again, cross-multiplying and solving for d, we get $4d = 5(n + 2)$, so $d = (5/4)(n + 2)$. Now, setting the two expressions for d equal to one another and solving yields $(5/3)n = (5/4)(n + 2) \rightarrow (4/3)n = n + 2 \rightarrow 4n = 3n + 6 \rightarrow n = 6$. That means that previously, there were 6 nurses on staff, and now, there are $n + 2 = 6 + 2 =$ **8** nurses.

Sprint 11

Working backward, we subtract 3 and then divide by 3 to find each previous term. The third term is 39. So, the second term is $(39 - 3)/3 = 12$, and the first term is $(12 - 3)/3 =$ **3**.

Sprint 12

The next day that Bill and Jill will both exercise will be in the number of days that is equal to the least common multiple (LCM) of 4 and 5. We can factor 4 and 5 as 2×2 and 5×1, respectively, and thus the LCM of 4 and 5 is $2 \times 2 \times 5 = 20$. Therefore, in 20 days they will exercise together again, which is January $1 + 20 =$ **January 21**.

Sprint 13

The square region created by these vertices is 6 units by 6 units, resulting in a 5-unit by 5-unit array of lattice points in the interior of this region. Thus, there are $5 \times 5 =$ **25** points with integer coordinates.

Sprint 14

There is only 1 way to make a four-bead bracelet with just red beads. Likewise, there is only 1 way to make a bracelet with all green beads. There is only 1 way to make a bracelet with three red beads and one green. Similarly, there is 1 way to make a bracelet with three green beads and one red. When we consider using two red beads and two green beads, however, we find that there are 2 possible bracelets (R-R-G-G or R-G-R-G). Thus, there are $1 + 1 + 1 + 1 + 2 =$ **6** different color patterns.

Sprint 15

Since Melton used 2.5 gallons of yellow paint in the sample and 30 gallons in the main batch, we know that the main batch is $30 \div 2.5 = 12$ times as large. Since Melton used $12 + 2.5 + 0.5 = 15$ gallons of paint for the sample, he used $15 \times 12 =$ **180** gallons of paint for the main batch.

Sprint 16

First, count up all the 1×1 squares. There are 6 of these squares, as shown in Figure 1. Now, count all the 2×2 squares. There are 2 of these, as shown in Figure 2. Finally, we can find 2 additional squares by counting the $\sqrt{2} \times \sqrt{2}$ squares, as shown in Figure 3. In total, there are $6 + 2 + 2 =$ **10** squares.

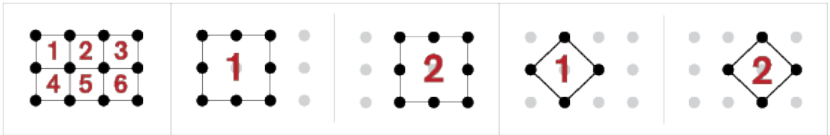


Figure 1 Figure 2 Figure 3

2021 Practice Competition 3 Solutions

Sprint 17

Since Xinran walks 3 mi/h uphill, it will take him 1/3 hour, or 1/3 × 60 = 20 minutes, to walk one mile uphill. Likewise, since he walks 4 mi/h on a flat surface, it will take him 1/4 hour, or 1/4 × 60 = 15 minutes, to walk one mile on the flat surface, and since he walks 5 mi/h downhill, it will take him 1/5 hour, or 1/5 × 60 = 12 minutes, to walk one mile downhill. Xinran's round trip will take 20 + 15 + 15 + 12 = **62** minutes.

Sprint 18

Let's assume the marked-up price was \$100. Then, the 40% discounted price and the wholesale price both would be 60% of the marked-up price, or 0.60 × 100 = \$60. The wholesale price was increased by 100 − 60 = \$40 to get to \$100. An increase of \$40 is 40/60 = 2/3 ≈ **67**% of the wholesale cost.

Sprint 19

If b is less than 20, then k can be 0, 1 or 2, making 0, 7 or 14 the resulting values of b, respectively. We don't need to worry about negative integers, since we're told that a is greater than 12 and a is less than b. This means that $12 < b < 20$. Considering this fact, we see that the only possible answer for b is 14, which is the result when k = **2**.

Sprint 20

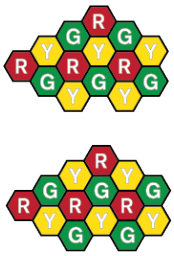
If one quart of paint is exactly enough for two coats of paint on a 9-foot by 10-foot wall, then one quart of paint covers 9 × 10 × 2 = 180 square feet. A 10-foot by 12-foot wall is 10 × 12 = 120 square feet. Thus, 120/180 = 12/18 = **2/3** quart is needed to cover a 10-foot by 12-foot wall.

Sprint 21

We can find the value of x by setting up the following equation: $45(0.1) + x(0.7) = 0.25(45 + x)$. Solving for x, we find that $4.5 + 0.7x = 11.25 + 0.25x \rightarrow 0.45x = 6.75 \rightarrow x =$ **15**.

Sprint 22

The figures show the **2** ways the figure can be colored red, yellow and green, so that no two hexagons with a common side are the same color.



Sprint 23

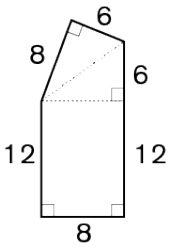
There are 5 × 5 = 25 combinations in total. Of these, the even products greater than 10 are 3 × 4 = 12, 4 × 3 = 12, 4 × 4 = 16, 4 × 5 = 20 and 5 × 4 = 20. Therefore, there are 5 different ways Josh can satisfy the requirements, so the probability that the product of the two numbers chosen will be even and greater than 10 is 5/25 = **1/5**.

Sprint 24

Let C = Claudia's age in 2019 and B = her brother's age in 2019. We can represent Claudia's age in 2019 as $C = 4B$. In 2020, we can represent her age as $C + 1 = 3(B + 1) \rightarrow C + 1 = 3B + 3 \rightarrow C = 3B + 2$. Now, we can set $3B + 2 = 4B$, as they're both equivalent to C, and solve for B to get $B = 2$. So, if Claudia's brother was 2 years old in 2019, then Claudia was 4 × 2 = 8 in 2019. Since the difference in their ages is 8 − 2 = 6 years, this means Claudia will be twice her brother's age when he is 6 years old and she is 12 years old. At 8 years old in 2019, Claudia will turn 8 + 4 = 12 in 2019 + 4 = **2023**.

2021 Practice Competition 3 Solutions

Sprint 25



Start by drawing a horizontal line parallel to the 8-inch base, as shown, dividing the original pentagon into a rectangle and a kite. The area of the rectangle is $12 \times 8 = 96$ in^2. Now, draw a segment joining the two non-right-angle vertices of the kite, and notice that the kite consists of two congruent right triangles. The area of the kite, then, is the sum of the areas of these two congruent right triangles, or $2 \times (1/2)(8)(6) = 48$ in^2. Therefore, the area of the pentagon is $96 + 48 =$ **144** in^2.

Sprint 26

First, we know there are $3 + 5 + 2 = 10$ marbles in total in the bag. We can note that the three marbles drawn from the bag could not all be blue, since there are only two blue marbles in the bag. So, in order for all three marbles to be the same color, they must be either all yellow or all red. The probability that the three marbles will be yellow is $(5/10) \times (4/9) \times (3/8) = 60/720$. Similarly, the probability that the three marbles will be red is $(3/10) \times (2/9) \times (1/8) = 6/720$. Therefore, the probability that all three marbles drawn will be the same color is $60/720 + 6/720 = 66/720 =$ **11/120**.

Sprint 27

Based on the provided information, we can write the follow equations: [1] pop = 1.25(peep), [2] slug = 0.6(pop) and [3] slap = 2(slug). Using what we know from equation [2], we can substitute the value of a slug into equation [3] to get slap = 2×0.6(pop) = 1.2(pop). Then, using equation [1], we can calculate that 1.2(pop) = 1.2×1.25(peep) = 1.5(peep). Therefore, a slap is $1.5 \times 100 =$ **150**% of a peep.

Sprint 28

Jonah's age has two digits, which we'll call x and y. We can represent Jonah's age with $10x + y$. Reversing this becomes $10y + x$, so $[(10y + x)/3] + 20 = 10x + y$. Multiplying both sides of this equation by 3, we get $10y + x + 60 = 30x + 3y$. Simplifying gives $7y + 60 = 29x$. Since x is a single digit, it must be large enough that $29x$ is greater than 60. Therefore, $x \geq 3$. If $x = 3$, then $29x = 87$ and $7y = 27$, which does not work. If $x = 4$, then $29x = 116$ and $7y = 56$, so $y = 8$. Therefore, Jonah must be $10x + y = 10 \times 4 + 8 = 40 + 8 =$ **48** years old.

Sprint 29

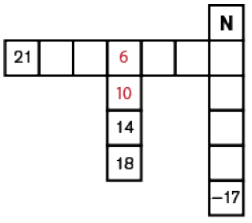


Figure 1

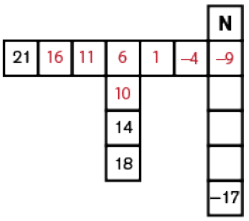


Figure 2

Let's start by looking at the column with 14 and 18. Because we know the values in this column form an arithmetic sequence, we know each value will differ by 4, so we can fill in the rest of this column as shown in Figure 1. Now, in the row beginning with 21, we now have 21, _, _, 6, where each value differs by the same number. If x is the difference between terms in the sequence, we get 21, $21 - x$, $21 - 2x$, $21 - 3x$, etc. Therefore, $21 - 3x = 6$. Solving for x gives $x = 5$, and so we can fill in the rest of this row as shown in Figure 2. Finally, we can look at the final sequence in the right-hand column, where we'll let y be the difference between terms in the sequence. So, we have N, $N + y$, $N + 2y$, $N + 3y$, $N + 4y$ and $N + 5y$. From these terms, we know that $N + 5y = -17$ and $N + y = -9$. Subtracting the second equation from the first equation gives $4y = -8$, so $y = -2$. Substituting this value into $N + y = -9$, we get $N - 2 = -9$, so $N =$ **−7**.

Sprint 30

Cross-multiplying, we can simplify the three equations to get [1] $4a = 3b$, [2] $9b = 8c$, and [3] $3c = 2d$. Let's rewrite equation [3] as $c = (2/3)d$. Then, we can substitute this value for c into equation [2] to get $9b = 8(2/3)d \rightarrow 9b = (16/3)d \rightarrow (27/16)b = d$. We can rewrite equation [1] to be $a = (3/4)b$. Therefore, $ad/b^2 = [(3/4)b \times (27/16)b]/b^2 = [(81/64)b^2]/b^2 =$ **81/64**.

2021 Practice Competition 3 Solutions

Target 1

Each year, 5% of the balance is added to the balance of Helga's account. The shortcut is to multiply by 1.05 for each year that she keeps the investment, since she will have 105% of what she had the previous year. Thus, after four years, Helga will have a balance of $\$1000 \times 1.05^4 \approx \$1216$. That means the amount of interest that Helga earned is $1216 - 1000 =$ \$**216** or \$**216.00** in interest.

Target 2

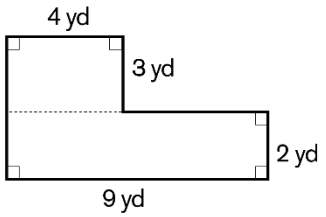
The sum of the first 10 positive multiples of 5 is $5 + 10 + 15 + \cdots + 40 + 45 + 50 = (50 + 5) \times 10 \div 2 = 275$. The sum of the first 10 positive, even integers is $2 + 4 + 6 + \cdots + 16 + 18 + 20 = (20 + 2) \times 10 \div 2 = 110$. Thus, the absolute difference between these sums is $275 \pm 110 =$ **165**.

Target 3

A stack of 70 wipes is 2 inches tall, so we need to determine the height of a stack of 100 wipes. Using the proportion $2/70 = h/100$, we see $200 = 70h$, so $h =$ **2.9** inches, to the nearest tenth.

Target 4

We can divide the region into two rectangles, as shown. One rectangle is 4 yards by 3 yards with area $4 \times 3 = 12$ yd^2, and the other rectangle is 9 yards by 2 yards with area $9 \times 2 = 18$ yd^2. The total area is $12 + 18 = 30$ yd^2. For carpet and padding, the cost per square yard is $21.95 + 2.55 = \$24.50$. So, for an area of 30 yd^2, the total cost will be $30 \times \$24.50 =$ \$**735** or \$**735.00**.



Target 5

If there were no restrictions on where anyone stands, there would be a total of $4! = 4 \times 3 \times 2 \times 1 = 24$ ways for the students to stand in a straight line. Let's label the students A, B, C and D, and let's assume that A and B refuse to stand next to each other. We must subtract those arrangements where students A and B are together. If we imagine A and B are a single unit AB, like a single student, then there are $3! = 3 \times 2 \times 1 = 6$ arrangements where they are together in this way. They could also be a single unit arranged as BA, so there are another $3! = 3 \times 2 \times 1 = 6$ arrangements where they are together. Since there are $6 + 6 = 12$ arrangements where A and B are together, that means there must be $24 - 12 =$ **12** ways for the students to stand in a straight line where A and B are not next to each other.

Target 6

First, let's rewrite the equation in terms of y to get $y = 20x^2 + 24$. We know that $20x^2$ will never be negative because x^2 will always be positive, and the product of two positive numbers is also positive. So, the least possible value for y would be when $x = 0$, and $y = 20 \times 0^2 + 24 =$ **24**.

Target 7

Let the current ages of Bill, Phil, Jenny and their dad be represented by B, P, J and D, respectively. We can write the following equations based on the provided information: [1] $B = 2P$, [2] $J = B - 2$ and [3] $D = 2(B + J + P)$. Let's simplify the third equation to get $D = 2B + 2J + 2P$ and substitute the first and second equations in to get $D = 2B + 2(B - 2) + B \rightarrow D = 2B + 2B - 4 + B \rightarrow D = 5B - 4$. Now, we can write a fourth equation to represent their dad's age in 9 years: $D + 9 = (B + 9) + (J + 9) + (P + 9) \rightarrow D + 9 = B + J + P + 27 \rightarrow D = B + J + P + 18$. If we rewrite the first equation as $P = (1/2)B$, we can substitute the first and second equations into the fourth equation to get $D = B + B - 2 + (1/2)B + 18 \rightarrow D = 2.5B + 16$. Now, we can set both equations for their dad's age equal to each other and solve: $5B - 4 = 2.5B + 16 \rightarrow 2.5B = 20 \rightarrow B = 8$. Therefore, if Bill is currently 8 years old, then Jenny is $8 - 2 =$ **6** years old.

Target 8

If we currently have 80% ammonia, then 4 gallons are ammonia, and 1 gallon is water. In quarts, this is 16 quarts of ammonia and 4 quarts of water. We want to drain out x quarts of the mixture and add in x quarts of water so that the mixture contains 10 quarts of water and 10 quarts of ammonia. So, $16 - 0.8x = 10$ and $4 - 0.2x + x = 10$. Setting these expressions equal to each other and simplifying, we see that $16 - 0.8x = 4 - 0.2x + x \rightarrow 16 - 4 = 0.8x - 0.2x + x \rightarrow 12 = 1.6x \rightarrow x = 7.5$. Therefore, **7.5** quarts must be drained from the mixture.

Team 1

The total time spent by the students to solve the problem is 30,000 × 24 = 720,000 seconds. One hour is equal to (60 minutes per hour) × (60 seconds per minute) = 3600 seconds. So, the ratio of the development time to the total time spent solving by the students is 3600/720,000 = **1/200**.

Team 2

Let's assume the polygon in question has n sides. The sum of the degrees of the interior angles of a polygon with n sides and n angles is $180(n - 2)$. Using the information provided in the problem, we can set up the equation $160 + [112(n - 1)] = 180(n - 2)$. Simplifying and solving for n gives us $160 + 112n - 112 = 180n - 360 \rightarrow 112n + 48 = 180n - 360 \rightarrow 68n = 408 \rightarrow n = 6$. Therefore, the polygon has **6** sides.

Team 3

Since all 20 legs must belong to the cows, who have four legs each, there must be 20 ÷ 4 = 5 cows. Next, since all 17 heads must belong to the cows and octopi, all of which have one head each, there must be 17 − 5 = 12 octopi. Finally, the 196 tentacles must belong to the octopi, who have eight tentacles, and the jellyfish, who each have 20 tentacles. So, there must be (196 − 12 × 8) ÷ 20 = 5 jellyfish. Thus, Farmer Brown has 5 + 12 + 5 = **22** animals.

Team 4

Let n = the number of nickels, d = the number of dimes and q = the number of quarters. Then, $n + d + q = 45$, $d = n + 1$ and $q = 3n - 1$. We can substitute the expressions for d and q from the second and third equations into the first to get $n + n + 1 + 3n - 1 = 45$. Simplifying and solving for n, we get $5n = 45$, and $n = 9$. Therefore, Keisha has **9** nickels.

Team 5

Using the distance formula, *distance* = *rate* × *time*, we can find that Joann traveled 12 mi/h × 3.5 hours = 42 miles. From the same formula, we know that *rate* = *distance* ÷ *time*, so we determine the rate at which Frank needs to travel to be *rate* = 42 miles ÷ 3 hours = **14** mi/h.

Team 6

First, there are 16 triangles that contain no smaller triangles within them, as shown in Figure 1. Next, there are 10 triangles that are made of 2 smaller triangles, as shown in Figure 2. There are no triangles made of 3 smaller ones, but there are 8 triangles made of 4 smaller ones, as shown in Figure 3. Finally, there are 2 triangles that are made of 8 smaller triangles, as shown in Figure 4. Thus, there are a total of 16 + 10 + 8 + 2 = **36** triangles.


Figure 1

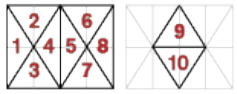


Figure 2

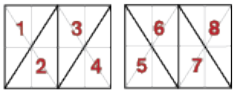


Figure 3

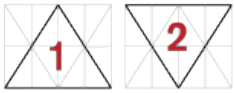


Figure 4

Team 7

Determine the positive integer divisors of 175 by first factoring to find $175 = 5 \times 5 \times 7$. Thus, the positive integer divisors of 175, besides 1, are 5, 7, 25, 35 and 175. Only two of these values are multiples of 7, namely 35 and 175, and thus would be adjacent to 7 around the circle. So, the sum of these two integers is $175 + 35 =$ **210**.

Team 8

Let's enumerate the number of perfect squares less than or equal to 200. They are 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169 and 196. So, if we removed these from the first 200 numbers, we would be left with 186 numbers. To account for these "missing" numbers, we add $200 + 14 = 214$. The next perfect square is 225, which is past 214, so the 200th term is **214**.

Team 9

We have $3 + 4 = 7$ non-yellow balls, so they must all be drawn first. There is a total of $3 + 4 + 5 = 12$ balls. So, the probability that the 7 non-yellow balls are drawn first is $(7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1)/(12 \times 11 \times 10 \times 9 \times 8 \times 7 \times 6) =$ **1/792**.

Team 10

Because the final exam counts as two tests, we can say that Heidi's semester average is made up of $6 + 2 = 8$ equally weighted tests. In order to have a 90-point average at the end of the semester, Heidi would need to have scored a total of $8 \times 90 = 720$ points. If she earns 99 points on the final exam (which counts as two tests), then on the other 6 tests, Heidi would have scored a total of $720 - (2 \times 99) = 522$ points. So, on average, she scored $522/6 =$ **87** points on each test prior to the final exam.